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Thermal Fields as Causal Fluids

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In a previous paper we investigated a divergence-type theory (DDT) describing the dissipative interaction between a field and a fluid. In this paper we compare the macroscopic view of DDT with a microscopic special case, an $O(N)$ scalar field to leading order in the large N approximation and its thermal fluctuations. Our aim is to compare within a simple model the two approaches.

KEY WORDS: divergence-type theory; inflation; preheating.

1. INTRODUCTION

In a previous paper we investigated a divergence-type theory (DDT; Geroch and Lindblom, 1990) describing the dissipative interaction between a field and a fluid (Calzetta and Thibeault, 2001). We looked for theories that, under equilibrium conditions, reduce to a Klein–Gordon (KG) scalar field and a perfect fluid. We showed that the requirements of causality and positive entropy production put nontrivial constraints on the structure of the interaction terms.

In this paper we compare the macroscopic view of DDT with a microscopic special case, an $O(N)$ scalar field to leading order in the large N approximation and its thermal fluctuations. Our aim is to compare within a simple model the two approaches. Our basic motivation is very simple: to understand a complex system, it is not always beneficial to work out the basic microscopic theory. The archetypal example is of course the description of a gas using Newton's laws on its microscopic constituents instead of thermodynamics. Another example that brings us closer to our discussion is the description of a viscous fluid. The Navier–Stokes equations give only a smoothed out, macroscopic description of the dissipative fluid, but more than sufficient for all practical purposes. Save in very specific cases, few will try to describe a fluid by going down to its molecular components and the quite complex interactions between them.

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The inflationary universe (Kolb and Turner, 1994) presents us with a similar challenge. There various out-of-equilibrium fields interact nonlinearly. The coupling with the metric makes things worse. Even if one works in the classical limit, numerous approximations are needed to obtain answers and it is not always clear if they do not hide the very behavior that we try to understand.

On the other side, if one could have a reliable macroscopic theory then the relevant behavior would reveal itself. Sweeping generalizations like homogeneity could be avoided and the system could be studied in its true complexity. However, we do not have the equivalent of Navier–Stokes equations here nor it is clear that a proper equivalent set of equations can even be obtained. First, one needs a suitable framework where relativistic fluids can be described. That is where DTT comes in. Second, we have to describe basic fields like the Klein–Gordon field in this framework. Third, we have to understand which basic parameters are sufficient to describe the system at hand. In this work, we will only test our models in a very simple context to see if the two approaches can be used in a meaningful way to describe the same physical system, the relevant variable being the mean value of the scalar field.

The rest of the paper is organized as follows. In the next section we present the theory of a scalar field with $O(N)$ invariance and show that under suitable approximations it can be reduced to a Hamiltonian system. In Section 3 we write down a simple, nonlinear DTT and use it to analyze the thermalization process. Finally, in Section 4 we present the comparison between the two approaches. We summarize the main conclusions in some brief final remarks.

2. CLASSICAL FIELD WITH TEMPERATURE FLUCTUATIONS

The starting point for our microscopic model is the $O(N)$ invariant action

$$S = N \int \left\{ -\frac{1}{2}\partial_\mu \varphi^i \partial^\mu \varphi^i - \frac{\lambda}{2}\left(\frac{1}{2}\varphi^i\varphi^i + \frac{m^2}{\lambda}\right)^2 \right\} d^4x \qquad (1)$$

The model was already used in this context (Boyanovsky, *et al.*, 1996; Cooper *et al.*, 1995) and has many nice features. It is customary to define a new field by adding the constraint

$$\frac{[\chi - \lambda(\varphi^i\varphi^i/2 + m^2/\lambda)]^2}{2\lambda}$$

We obtain a new action S_2

$$S_2 = N \int \left\{ -\frac{1}{2}\partial_\mu \phi^i \partial^\mu \phi^i + \frac{\chi^2}{2\lambda} - \frac{1}{2}\chi\phi^i\phi^i - \frac{m^2}{\lambda}\chi \right\} d^4x$$

Representing the expectation value of the field with respect to the initial state of the theory by

$$\langle\varphi\rangle = \phi, \qquad \langle\chi\rangle = K$$

we can now write the fields as the sum of mean fields and fluctuations

$$\varphi^i = \phi^i + \psi^i \tag{2}$$
$$\chi = K + \kappa$$

Keeping only next to leading terms in the large N aproximation we can write the effective action

$$\Gamma[\phi^i, K] = S_2[\phi^i, K] + i\frac{N\hbar}{2}\,\mathrm{Tr}\,\ln\left(\frac{i}{2\hbar}\right)(\partial_\mu\partial^\mu - K)$$
$$= S_2\left[\phi^i, K\right] + i\frac{N\hbar}{2}\,\mathrm{Tr}\,\ln\left(\frac{-1}{2}\right)G_F^{-1}(x-z) \tag{3}$$

Variation of the effective action gives the equation for the classical field,

$$-\partial_\mu\partial^\mu\phi^i + K\phi^i = 0 \tag{4}$$

and for the K field,

$$K = m^2 + \frac{\lambda}{2}\phi^i\phi^i + N\lambda G_F(x,x) \tag{5}$$

It is helpful at this point to rotate in the internal space so that $\phi^i = 0$, $i \in \{1, N-1\}$, and $\phi^N \equiv \phi = \sqrt{\phi^i\phi^i}$. Formally, we thus have the equations for one mean field ϕ and its fluctuations save that the propagator is now multiplied by N. The Feynman propagator for the fluctuations is given by

$$G_F(x,x')\delta^{ij} = \langle T(\psi^i(x)\psi^j(x'))\rangle \tag{6}$$

which is solution of the following equation:

$$(-\partial_\mu\partial^\mu + m^2)G_F(x,x') = -i\hbar\delta(x-x')$$

where ψ^i represents a generic quantum fluctuating field. If we assume a homogeneous initial state, it is convenient to introduce the Fourier expansion

$$\psi^i(x) = \int\frac{d^3k}{(2\pi)^3}\,\exp(i\vec{k}\cdot\vec{x})\sqrt{\frac{\hbar}{2\omega_k(0)}}\left\{U_k(t)a^i_{\vec{k}} + U^*_k(t)a^i_{-\vec{k}}\right\} \tag{7}$$

The normalization for the modes is

$$W[U^*_k(t), U_k(t)] = -i\hbar \tag{8}$$

$W[f,g] = f\dot{g} - \dot{f}g$ being the Wronskian. In the homogeneous case,

$$\frac{d^2\phi}{dt^2} + K\phi = 0 \tag{9}$$

with the following equations of motion for the mode function $U_k(t)$:

$$\frac{d^2U_k(t)}{dt^2} + \omega_k^2(t)U_k(t) = 0; \quad \omega_k^2(t) = |\vec{k}|^2 + K(t) \tag{10}$$

Substituting (7) in (6) gives

$$G_F(x, x') = G_V(x, x') + G_T(x, x')$$

where we recognize a temperature-independent (or vacuum) part

$$G_V(x, x') = \int \frac{d^3k}{(2\pi)^3} \exp[i\vec{k}\cdot(\vec{x} - \vec{x}')]\{U_k(t)U_k^*(t')\Theta(t - t') + U_k(t')U_k^*(t)\Theta(t' - t)\}$$

and a temperature-dependent part

$$G_T(x, x') = \int \frac{d^3k}{(2\pi)^3}\{\exp[i\vec{k}\cdot(\vec{x} + \vec{x}')][U_k(t)U_k(t')g_k + U_k^*(t)U_k^*(t')g_k^*] + \exp[i\vec{k}\cdot(\vec{x} - \vec{x}')][U_k(t)U_k^*(t') + U_k^*(t)U_k(t')]n_k\}$$

where n_k and g_k represent the initial statistical mixture:

$$\langle a^i_{\vec{k}} a^j_{\vec{k}'}\rangle = (2\pi)^3 g_k \delta^3(\vec{k} - \vec{k}')\delta^{ij}$$

$$\langle a^{i\dagger}_{\vec{k}} a^j_{\vec{k}'}\rangle = (2\pi)^3 n_k \delta^3(\vec{k} - \vec{k}')\delta^{ij}$$

In the coincidence limit,

$$G_F(x, x) = G_V(x, x) + G_T(x, x)$$

with

$$G_V(x, x) = \int \frac{d^3k}{(2\pi)^3}\,|U_k(t)|^2$$

and

$$G_T(x, x) = \int \frac{d^3k}{(2\pi)^3}\,\{2\exp(i2\vec{k}\cdot\vec{x})\,\mathrm{Re}[U_k(t)U_k(t)g_k] + 2|U_k(t)|^2 n_k\}$$

Let us now suppose that initially we have a thermal bath of particles

$$n_k = \frac{1}{\exp\left(\frac{\hbar\omega_k(0)}{k_B T}\right) - 1}$$

$$g_k = 0$$

and take the limit $\hbar \to 0$. In this way quantum effects disappear. However the system does not become trivial since we still have thermal fluctuations. Indeed

$$\lim_{\hbar\to 0} G_V(x,x) = \lim_{\hbar\to 0} \int \frac{d^3k}{(2\pi)^3} \frac{\hbar}{2\omega_k(0)} |U_k(t)|^2 = 0$$

But

$$\lim_{\hbar\to 0} G_T(x,x) = \lim_{\hbar\to 0} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k(0)} \frac{\hbar}{\exp\left(\frac{\hbar\omega_k(0)}{k_B T}\right) - 1} |U_k(t)|^2$$

$$= \int \frac{d^3k}{(2\pi)^3} \frac{k_B T}{2\omega_k^2(0)} |U_k(t)|^2$$

Thus, in this classical limit, we find

$$K = m^2 + \frac{1}{2}\lambda\phi^2 + \lambda N k_B T \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k^2(0)} |U_k(t)|^2 \tag{11}$$

The energy–momentum tensor can be computed using (3) and

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta\Gamma}{\delta g_{\mu\nu}}$$

specializing thereafter to Minkowski or directly by taking the expectation value of the classical energy–momentum tensor. Either way, we find (writing only the nontrivial components)

$$\langle T^{00}\rangle = \frac{1}{2}\dot{\phi}^2 + k_B T \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k^2(0)} [\dot{U}_k(t)\dot{U}_k^*(t) + |\vec{k}|^2 |U_k(t)|^2]$$
$$+ \frac{1}{2\lambda}(K - \eta m^2)(K + \eta m^2) + \frac{m^4}{2\lambda}$$

and

$$\langle T^{ii}\rangle = \frac{1}{2}\dot{\phi}^2 + k_B T \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k^2(0)} \left[\dot{U}_k(t)\dot{U}_k^*(t) - \frac{1}{3}|\vec{k}|^2 |U_k(t)|^2\right]$$
$$- \frac{1}{2\lambda}(K - m^2)(K + m^2) - \frac{m^4}{2\lambda}$$

These expressions are the total energy density and pressure for our system. To integrate numerically our equations we need a finite number of variables. Since the integrands are manifestly isotropic, we perform first the integration over the angular variables, reducing the triple integral to a single one over the variable $k = |\vec{k}|$. In problems with spherical symmetry like this one, this procedure leads to a better approximation of the integrals than approximating the 3-D integral as a triple sum over Cartesian coordinates. The remaining integral is written as a finite sum:

$$\langle T^{00}\rangle = \frac{1}{2}\dot{\phi}^2 + Nk_{\rm B}T\sum_{k=1}^{N_k}\Delta k\alpha_k[\dot{U}_k(t)\dot{U}_k^*(t) + |\vec{k}\Delta k|^2|U_k(t)|^2] + \frac{1}{2\lambda}(K-m^2)(K+m^2) + \frac{m^4}{2\lambda} \quad (12)$$

and

$$\langle T^{ii}\rangle = \frac{1}{2}\dot{\phi}^2 + Nk_{\rm B}T\sum_{k=1}^{N_k}\Delta k\alpha_k\left[\dot{U}_k(t)\dot{U}_k^*(t) - \frac{1}{3}|\vec{k}\Delta k|^2|U_k(t)|^2\right] - \frac{1}{2\lambda}(K-m^2)(K+m^2) - \frac{m^4}{2\lambda} \quad (13)$$

where we defined

$$\alpha_k \equiv \frac{|\vec{k}|^2}{4\pi^2\omega_k^2(0)}$$

and we set a cutoff frequency $k_{\rm max} = N_k\Delta k$. It is a straightforward exercise to verify that we can now rewrite the whole system as a Hamiltonian system (H has units of energy density). Writing the complex modes $U_k(t)$ as real and imaginary parts

$$U_k(t) = U_r(t) + iU_i(t)$$

and writing $p \equiv \dot{\phi}$ the Hamiltonian is

$$H = \frac{1}{2}p^2 + \frac{1}{2}K\phi^2 + \sum_{k=1}^{N_k}\left\{\frac{1}{4Nk_{\rm B}T\alpha_k\Delta k}\left[\Pi_r^2 + \Pi_i^2\right] + Nk_{\rm B}T\alpha_k\Delta k(|\vec{k}|^2 + K)\left[U_r^2 + U_i^2\right]\right\} - \frac{1}{2\lambda}(K-m^2)^2 \quad (14)$$

The equations of motion are

$$-\dot{\Pi}_r = \frac{\partial H}{\partial[U_r]} = 2Nk_{\rm B}T\alpha_k\Delta k(|\vec{k}|^2 + K)U_r$$

$$\dot{U}_r = \frac{\partial H}{\partial[\Pi_r]} = \frac{1}{2Nk_{\rm B}T\alpha_k\Delta k}\Pi_r$$

and

$$-\dot{\Pi}_i = \frac{\partial H}{\partial[U_i]} = 2Nk_{\rm B}T\alpha_k\Delta k(|\vec{k}|^2 + K)U_i$$

$$\dot{U}_i = \frac{\partial H}{\partial[\Pi_i]} = \frac{1}{2Nk_{\rm B}T\alpha_k\Delta k}\Pi_i$$

which also gives the definitions of Π_r and Π_i. Initially, we have $U_r(0) = 1, \dot{U}_r = 0$ and $U_i(0) = 0, \dot{U}_i = -\omega_k(0)$.

K is a cyclic variable and its corresponding Hamiltonian equation gives the (discretized) gap equation (11). At $t = 0$ this reads

$$\begin{aligned} K(0) &= m^2 + \frac{\lambda}{2}\phi^2(0) + \lambda N k_{\rm B} T \sum_{k=1}^{N_k} \Delta k \alpha_k \\ &= m^2 + \frac{\lambda}{2}\phi^2(0) + \frac{\lambda N k_{\rm B} T \Delta k}{4\pi^2} \sum_{k=1}^{N_k} \frac{|\Delta k \vec{k}|^2}{(|\Delta k \vec{k}|^2 + K(0))} \end{aligned} \tag{15}$$

an equation to be solved numerically to extract $K(0)$ as a function of the initial conditions and parameters.

3. FLUIDS

In a previous paper (Calzetta and Thibeault, 2001) we developed a consistent relativistic framework to describe the mixture of scalar fields and perfect fluids in the general context of DTT (Geroch and Lindblom, 1990). In covariant language, a perfect fluid is a system whose energy–momentum tensor takes the form $T^{ab} = g^{ab} + u^a u^b(\rho + p)$, where ρ is the energy density as seen by an observer moving with the fluid, p is the pressure, g^{ab} is the metric, and u^a is the 4-velocity. Usually this is not sufficient to characterize completely the fluid, and another equation appears in the form of a conserved current $j^a_{;a} = 0$, where $j^a = ju^a$, j being the corresponding density as seen by a comoving observer. In DTT, T^{ab} and j^a are assumed to be derivable from a generating function χ^a

$$T^{ab} = \frac{\partial \chi^a}{\partial \xi_b}, \qquad j^a = \frac{\partial \chi^a}{\partial \xi}$$

ξ and ξ^a representing now the dynamical degrees of freedom of the theory. χ^a can be further simplified since, as a consequence of the symmetry of T^{ab}, we have

$$\chi^a = \frac{\partial \chi}{\partial \xi_a} \tag{16}$$

That is, all the fundamental tensors of the theory can be obtained from the generating functional χ. A perfect fluid is obtained if $\chi = \chi(\xi, \mu)$, where $\mu \equiv \sqrt{-\xi^a \xi_a}$. The Klein–Gordon theory can be obtained using

$$\chi_c = -\frac{1}{2}\xi^2 \ln \beta_{(c)} + \frac{1}{2} V \beta_{(c)}^2$$

where $\beta_{(c)}$, is the inverse temperature, the current $j^a = \frac{\xi}{\beta^2_{(c)}}\beta^a_{(c)}$, and $\xi = -\beta^a\phi_a$. The Klein–Gordon equation is given by $j^a_{;a} = V'(\phi)$, $V(\phi)$ being the potential. The theory describing the scalar field and a perfect fluid (q-fluid) together can be obtained enlarging the generating functional by the addition of an interaction functional Ξ (Ξ^a being defined as in (16). The energy–momentum tensors of field and fluid will not be individually conserved and furthermore the Klein–Gordon equation will also deviate from its original form. The total set of equations governing our theory can be written as

$$\begin{aligned} j^a_{;a} &= V' + \Delta \\ T^{ab}_{(c);b} &= I^a \\ T^{ab}_{(q);b} &= -I^a \\ \beta^a_{(c)}\phi_{,a} &= -\xi \end{aligned} \tag{17}$$

j^a and T^{ab}_c are the current and energy–momentum tensor for the scalar field and

$$T^{ab}_{(q)} = \frac{\partial(\chi^a + \Xi^a)}{\partial\beta_{(q)b}}$$

The pressure is by definition given by

$$\Pi \equiv T^{ii}$$

In the homogeneous case, the only nontrivial, independent equation for the energy–momentum tensor is the $a = b = 0$ one. It is convenient to work with

$$T^{00}_+ = T^{00}_{(c)} + T^{00}_{(q)} = \frac{\partial\chi^a}{\partial\beta_b} \tag{18}$$

$$T^{00}_- = T^{00}_{(c)} - T^{00}_{(q)} = 2\frac{\partial\chi^a}{\partial B_b} \tag{19}$$

where the new variable $B^a = \beta^a_{(c)} - \beta^a_{(q)}$ and $2\beta = \beta^a_{(c)} + \beta^a_{(q)}$. To compute (18) and (19) we need to propose some specific form for the interaction functional Ξ. We use as model for our interaction term (Calzetta and Thibeault, 2001)

$$\Xi = f_0\frac{v}{u^3} + g_0\frac{w^2}{u^3} \tag{20}$$

where $u = \sqrt{-\beta^a\beta_a}$, $w = \sqrt{-B^a\beta_a}$, and $v = \sqrt{-B^aB_a}$ are the scalars that can be constructed from the temperature vectors B^a and β^a. Therefore the system, restricted to the homogeneous case:

$$-p_{,t} = V'(\phi) + \Delta \tag{21}$$

$$T^{00}_{+,t} = 0 \tag{22}$$

$$T^{00}_{-,t} = 2I^0 \tag{23}$$

$$\phi_{,t} = p \tag{24}$$

Note that the total energy T^{00} is conserved [Eq. (22)]. In the following it is convenient to work with the variable s defined from

$$B = \beta_c - \beta_q \equiv s\beta_q$$

and $\beta_{(q)} \equiv \beta_q$. We still have to define the q-fluid. Since it is a classical fluid, we choose an equation of state consistent with the classical equipartition theorem

$$p_q = \frac{a_c n}{3\beta_q}$$

$$\rho_q = \frac{a_c n}{\beta_q}$$

with $a_c = 3$, and n being the particle density of the fluid. We can now compute explicitly T^{00}_+ and T^{00}_- using (18)–(20)

$$T^{00}_+ = \frac{1}{2}p^2 + V(\phi) + \frac{a_c n}{\beta_q} + 20\Gamma\frac{s^2}{(1+s/2)^6}\frac{1}{\beta_q^4} \tag{25}$$

$$T^{00}_- = \frac{1}{2}p^2 + V(\phi) - \frac{a_c n}{\beta_q} - 16\Gamma\frac{s}{(1+s/2)^5}\frac{1}{\beta_q^4} \tag{26}$$

where we introduce $\Gamma = f_0 + g_0$.

Now we must model the right-hand side. In this context, the entropy flux is given by $S^a = \chi^a - T^{ab}_+\beta_b - T^{ab}_- B_b - j^a\xi$ and the entropy creation

$$\nabla_a S^a = -B_b I^b - \xi\Delta \tag{27}$$

Within leading order in the large N approximation the entropy creation is exactly zero and we must have

$$\Delta = M\beta B \tag{28}$$

$$I^a = M\xi\beta u^a \tag{29}$$

We will be interested particularly in this special case since we want our fluid to be compared with a Hamiltonian, nondissipative theory. M has dimensions of temperature to the fifth power and so we can write

$$M = \frac{\kappa_0}{\tau^5} \tag{30}$$

with κ_0 being a dimensional constant and τ is a function of the basic fluid variables with units of inverse temperature $(k_B T)^{-1}$. Using (28)–(30) we write

$$\Delta = \frac{\kappa_0}{\tau^5}\left(1 + \frac{s}{2}\right)s\beta_q^2 \tag{31}$$

$$I^a = -\frac{\kappa_0}{\tau^5} p(1+s)\left(1+\frac{s}{2}\right)\beta_q^2 \tag{32}$$

A choice must be made for τ. We choose

$$\begin{aligned}\tau &= \frac{1}{|\xi|}\\ &= \frac{1}{\beta_c p} = \frac{1}{(1+s)\beta_q p}\end{aligned} \tag{33}$$

We can now turn to the task of obtaining explicitly the equations of motions for the fluid. We introduce

$$d_1(s) \equiv 16\Gamma\frac{s}{(1+s/2)^5} \tag{34}$$

$$d_2(s) \equiv 20\Gamma\frac{s^2}{(1+s/2)^6} \tag{35}$$

Substituting (25) and (26) into (22) and (23) respectively, we can write the differential equations for s and β_q as

$$-\Delta(t)p(t) - a_c n\frac{1}{\beta_q^2}\beta_{q,t} + d_2'(s)\frac{1}{\beta_q^4}s_{,t} - 4d_2(s)\frac{1}{\beta_q^5}\beta_{q,t} = 0$$

$$-\Delta(t)p(t) + a_c n\frac{1}{\beta_q^2}\beta_{q,t} - d_1'(s)\frac{1}{\beta_q^4}s_{,t} + 4d_1(s)\frac{1}{\beta_q^5}\beta_{q,t} = 2I^0$$

It is a straightforward exercise to obtain equations for s and β. The equations of motion for the dissipative fluid are thus

$$s_{,t} = \frac{2\{(a_c\beta_q^3 + 4d_2(s))I^0 + [a_c\beta_q^3 + 2(d_1(s) + d_2(s))]\Delta(t)p(t)\}}{4W[d_1(s), d_2(s)] - a_c\beta_q^3(d_1'(s) - d_2'(s))}\beta_q^4 \tag{36}$$

$$\beta_{q,t} = \frac{2d_2'(s)I^0 + (d_1'(s) + d_2'(s))\Delta(t)p(t)}{4W[d_1(s), d_2(s)] - a_c\beta_q^3(d_1'(s) - d_2'(s))}\beta_q^5 \tag{37}$$

$$\phi_{,t} = p \tag{38}$$

$$p_{,t} = -V'(\phi) - \Delta \tag{39}$$

where Δ, I^0, d_1, d_2, and $W[a, b]$ are defined in (31), (32), (34), (35), and (8) respectively. We choose

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{8}\lambda\phi^4$$

as in (1).

4. COMPARISON BETWEEN THE THERMAL MODEL AND THE RELATIVISTIC FLUID

We want to compare numerically the theory described by (14) with the one described by (36)–(39). In the first one we have $4N_k + 2$ ordinary differential equations, and in the second, 4. The parameters that are common to both are m, the mass of the scalar partícle, and λ, the coupling parameter of the ϕ^4 theory. They will be set equal in each set. On the Hamiltonian side we have as parameters N, N_k, Δk, T, and on the fluid side k_0, f_0, g_0, n. Since the role of the q-fluid is to describe the thermal bath, the initial conditions for $\beta_q(0) = (k_B T)^{-1}$, the inverse temperature of the thermal bath of the fluctuations. Initially the Hamiltonian density and the total pressure of the microscopic model could be written as

$$\begin{aligned} H = {} & \frac{1}{2} p_0^2 + \frac{1}{2} K(0)\phi_0^2 + \frac{N k_B T}{12\pi^2} (\Delta k)^3 N_k (N_k + 1)(2N_k + 1) \\ & - \frac{(K(0) - m^2)^2}{2\lambda} + \frac{m^2}{2\lambda} \end{aligned} \tag{40}$$

$$\begin{aligned} \langle T^{ii} \rangle = {} & \frac{1}{2} p_0^2 - \frac{1}{2} K(0)\phi_0^2 + \frac{N k_\mathrm{B} T}{36\pi^2} (\Delta k)^3 N_k (N_k + 1)(2N_k + 1) \\ & - \frac{(K(0) + 3m^2)(K(0) - m^2)}{6\lambda} - \frac{m^2}{2\lambda} \end{aligned} \tag{41}$$

with $K(0)$ being the solution of the implicit equation (15). In the other model we have

$$T_+^{00} = \frac{1}{2} p_0^2 + V(\phi_0) + \frac{a_c n}{\beta_q(0)} + 20(f_0 + g_0) \frac{s_0^2}{(1 + s_0/2)^6} \frac{1}{\beta_q^4(0)} \tag{42}$$

$$\Pi_+ = \frac{1}{2} p_0^2 - V(\phi_0) + \frac{a_c n}{3\beta_q(0)} + 2(2 f_0 + 3g_0) \frac{s_0^2}{(1 + s_0/2)^6} \frac{1}{\beta_q^4(0)} \tag{43}$$

where Π_+ denotes the total pressure in the fluid model. We demand that initially the total energy density and pressure be equal. For fixed s_0 and f_0, this is a system of two equations for two unknowns n and g_0. For the simulation that is presented here, $f_0 = -1.1$ and $s_0 = 0.2$. The initial conditions for the scalar mean field and its conjuguate momenta were chosen equal: $\phi_0 = 2.2$ and $p = 0$. We obtain $n = 20.05$ and $g_0 = 178$. The value for k_0 was 0.02. N was set to 10 and the cutoff $N_k = 160$ with $\Delta k = 0.1$. The mass $m = 2.2$ and $\lambda = 0.75$.

Both systems of equations (DTT and thermal fluctuations) were integrated using Burlich–Stoer adaptive routine (Vetterling *et al.*, 1993). This is both faster and more precise than the more familiar Runge–Kutta adaptive step routine, which was nevertheless used to verify the numerical output of our routine. Since both

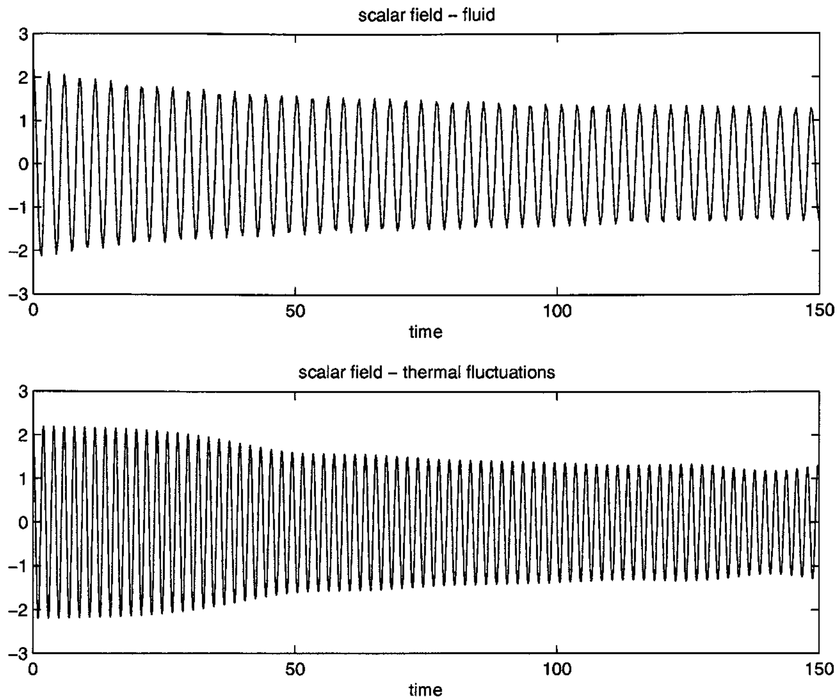

Fig. 1. Time dependence of the scalar field–fluid and the scalar field–thermal fluctuations.

systems conserve energy, as a consistency check for the precision of the numerical integration we computed the energy density in both cases and verified that it remains constant within acceptable tolerance (usually less than one part in a thousand for the thermal fluctuations; in the case of the fluid, the Hamiltonian was conserved down to numerical precision $\sim 10^{-15}$) (Fig. 1).

5. CONCLUSIONS

We used the macroscopic view of DDT in the case of a scalar field and a fluid to compare it with a microscopic special case, an $O(N)$ scalar field to leading order in the large N approximation and its thermal fluctuations.

We found that we can describe quite well the damping of the scalar field, which is the crucial element in undertanding the cosmological reheating process. On the other hand, the frequency of oscillations in the microscopic model tends to be higher than in the macroscopic one. This difference can be attributed to our simplifying assumption of equating the $V(\phi)$ potential in both models (in the macroscopic model, an effective potential should be used) and the naive linear equation of state for the fluid.

The success in describing the damping of the mean field is an indication that this is a promising approach for future research in reheating and thermalization in cosmology.

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REFERENCES

Boyanovsky, D., de Vega, H. J., Holman, R., and Salgado, J. F. J. (1996). analytic and numerical study of preheating dynamics, *Physical Review D: Particles and Fields* **54**(12), 7570.

Calzetta, E. and Thibeault, M. (2001). Relativistic theories of interacting fields and fluids, *Physical Review D: Particles and Fields* **63**, 103507.

Cooper, F., Kluger, Y., Mattola, E., and Paz, (1995). Quantum evolution of disoriented chiral condenstes, *Physical Review D: Particles and Fields* **51**, 2377.

Geroch, R. and Lindblom, L. (1990). Dissipative relativistic fluid theories of divergent type, *Physical Review D: Particles and Fields* **14**(6), 1855.

Kolb, E. W. and Turner, M. S. (1994). *The Early Universe*, pp. 261–317. Addison-Wesley, Reading, MA.

Vetterling, W., Teukolsky, S., Press, W., and Flannery, B. (1993). *Numerical Recipes*, pp. 718–725. Cambridge University Press, London.